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SPECIFICATION

TWO VARIABLE DATA INTERPOLATING SYSTEM

BACKGROUND OF THE INVENTION

Field of the Invention

The present invention relates to a two variable data interpolating system interpolating a value between discrete data positioned in a two-dimensional space. In this specification, it is assumed that a case where function values have finite values except zero in a local region and become zero in regions different from the region is called a "local support."

Description of the Prior Art

Conventionally, a method of performing data interpolation by using a sampling function is known as a data interpolation for obtaining a value between sample values that are given beforehand.

FIG. 7 is an explanatory diagram of a sampling function called a sinc function conventionally known. This sinc function is obtained when a Dirac delta function is inverse-Fourier-transformed, and becomes one only at a sample point, where t = 0, and zero at all other sample points. Concretely, let a sampling frequency be f, and the sinc function is expressed as follows:

$$x(t) = \sum_{k=-\infty}^{\infty} x(kT) \frac{\sin \pi f(t - kT)}{\pi f(t - kT)} \qquad \ldots (1)$$

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According to this equation (1), it can be seen that interpolation by the sinc function is realized by shifting a function of sin $\{\pi f(t - kT)\}/\pi f(t - kT)$ by kT in the direction of the time base, and multiplying the function by a sample value and adding them, that is, performing so-called convolution operation.

ins a1 ~~FIG. 8 is an explanatory diagram of data interpolation by using the sampling function shown in FIG. 7. As shown in FIG. 10, values except each sample point are interpolated by using all the sample values.~~

In addition, it is also possible to perform interpolation of two variable data such as an image by using the data interpolation system described above. Well-known conventional methods used for interpolation processing of image data are a nearest interpolation, a conjugate linear interpolation, a third convolution interpolation, and the like.

For example, so as to obtain a value of image data to be interpolated, let discrete data, which includes each two pixels before and after a data interpolating position in the x and y directions respectively by using the third convolution interpolation, be P_{11}, P_{12}, and the like, and the value of the interpolation data, P is calculated by:

$$P = [f(y_1)\, f(y_2)\, f(y_3)\, f(y_4)] \begin{bmatrix} P_{11} P_{21} P_{31} P_{41} \\ P_{12} P_{22} P_{32} P_{42} \\ P_{13} P_{23} P_{33} P_{43} \\ P_{14} P_{24} P_{34} P_{44} \end{bmatrix} \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \\ f(x_4) \end{bmatrix} \quad \ldots (2)$$

Here, f(t) is:

$$f(t) = \frac{\sin \pi t}{\pi t} \approx \begin{cases} 1 - 2|t|^2 + |t|^3 & (0 \leq |t| < 1) \\ 4 - 8|t| = 5|t|^2 - |t|^3 & (1 \leq |t| < 2) \\ 0 & (2 \leq |t|) \end{cases} \quad \ldots (3)$$

This is obtained by approximating the above-described sinc function with a cubic function.

By the way, in case of using the sinc function as a sampling function, it is theoretically possible to obtain an accurate interpolation value by adding values of respective sampling functions, corresponding to sample points from $-\infty$ to $+\infty$, with convolution. Nevertheless, when the above-described interpolation operation is actually attempted with one of various types of processors, a truncation error arises due to the truncation of processing within a finite interval. Therefore, this system has a problem that sufficient accuracy cannot be obtained if the interpolation operation is performed with a small number of sample values.

For example, in case of using the cubic convolution interpolation shown in the equation (2), an error becomes large because of not only approximating the sinc function with a cubic function in order to simplify the calculation but also performing calculation by forcibly assuming that pixels separating by two pixels or more do not affect the data interpolating position. In addition, as it can be seen from equation (2), calculation in the x direction and calculation in the y direction are performed separately, and influence of pixels in oblique directions is not considered. Since, actually, it is considered that the pixels in the oblique

directions also affect an interpolation position similarly to influence of pixels existing in the horizontal direction (x direction) and vertical direction (y direction), an error corresponding to that is included in a value of interpolation data obtained regardless of the influence of the pixels in the oblique directions.

SUMMARY OF THE INVENTION

The present invention is created in consideration of these points, and an object of the present invention is to provide a two variable data interpolation system that can reduce operation quantity and has a small error.

A two variable data interpolation system of the present invention performs interpolation operation between discrete data positioned at equal intervals in a two-dimensional space by using a sampling function that can be differentiated and has values of a local support. Therefore, since it is good enough only to make discrete data, included in this local support interval, be objects of the interpolation operation, the operation quantity is few, and it is possible to obtain good interpolation accuracy because of no truncation error arising.

In particular, it is preferable to use a function of the local support, which can be differentiated only once over the whole range, as the sampling function described above. It is considered that it is necessary that various signals existing in the natural world have differentiability because the signals change smoothly. Nevertheless, it is considered

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that it is not necessary that the differentiability is not always infinite, and that it is possible to sufficiently approximate natural phenomena so long as the signals can be differentiated only once.

In this manner, although there are many advantages by using a sampling function of the local support that can be differentiated finite times, conventionally, it was considered that a sampling function fulfilling these conditions did not exist. Nevertheless, by the present inventor's research, a function fulfilling the conditions described above is found.

Concretely, letting a third order B spline function be F(t), a sampling function H(t) to which the present invention is applied can be obtained by equation, $-F(t + 1/2)/4 + F(t) - F(t - 1/2)/4$. This sampling function H(t) is a function of a local support that can be differentiated only once in the whole region and whose value converges to zero at $t = \pm 2$, and fulfills two conditions described above. By performing the interpolation between discrete data by using such a function H(t), it is possible to perform the interpolation operation whose operation quantity is few and whose accuracy is high. Therefore, in case of using, for example, image data existing in a two-dimensional space as discrete data, it becomes possible to perform real-time processing whose accuracy is high.

In addition, the third order B spline function F(t) can be expressed as $(4t^2 + 12t + 9)/4$ in $-3/2 \le t < -1/2$, $-2t^2$

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$+ 3/2$ in $-1/2 \leq t < 1/2$, and $(4t^2 - 12t + 9)/4$ in $1/2 \leq t < 3/2$. Therefore, it is possible to perform calculation of the sampling function, described above, by such a piecewise polynomial expressed in quadric functions. Hence, it is possible to reduce the operation quantity due to comparatively simple operation contents.

In addition, it is possible to express the sampling function in quadric piecewise polynomials without expressing the sampling function by using the B spline function as described above. Concretely, it is possible to perform the above-described interpolation processing by using a sampling function defined in $(-t^2 -4t -4)/4$ in $-2 \leq t < -3/2$, $(3t^2 + 8t + 5)/4$ in $-3/2 \leq t < -1$, $(5t^2 + 12t + 7)/4$ in $-1 \leq t < -1/2$, $(-7t^2 + 4)/4$ in $-1/2 \leq t < 1/2$, $(5t^2 - 12t + 7)/4$ in $1/2 \leq t < 1$, $(3t^2 - 8t + 5)/4$ in $1 \leq t < 3/2$, and $(-t^2 + 4t - 4)/4$ in $3/2 \leq t \leq 2$.

In addition, the two variable data interpolation system of the present invention includes discrete data extracting unit, sampling function operating unit, and convolution operating unit so as to perform the above-described interpolation operation. The discrete data extracting unit extracts a plurality of discrete data, which exists within a predetermined range around a data interpolating position with sandwiching the data interpolating position, and which is an object of interpolation operation. The sampling function operating unit calculates a value of the sampling function $H(t)$ for each of a plurality of discrete data extracted in this manner, letting distance between the data interpolating

position and discrete data be t. Furthermore, the convolution operating unit performs convolution operation to a plurality of values of the sampling function that is obtained by the calculation. In this manner, just by calculating values of the sampling function corresponding to a plurality of discrete data extracted and performing the convolution operation to this result, it is possible to perform data interpolation between discrete values and to drastically reduce processing volume necessary for interpolation processing. Furthermore, since no truncation error arises by using a sampling function of a local support as described above, it is possible to increase processing accuracy. In addition, since values of the sampling function are calculated for all the discrete data included in a predetermined range around a data interpolating position, an interpolation error can be reduced by equally treating discrete data affecting the data interpolating position.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a block diagram showing the configuration of a data processor of this embodiment;

FIG. 2 is a graph showing a rage of pixel data extracted around a data interpolating position;

FIG. 3 is an explanatory graph of a sampling function used in operation in a sampling function operating section;

FIG. 4 is an explanatory diagram of calculation of distance between a data interpolating position and each pixel;

FIG. 5 is a graph showing a concrete example of calculating a value of the sampling function at the data interpolating position by allowing the sampling function to correspond to each pixel;

FIG. 6 is an explanatory graph in the case where intervals where values of the sampling function become zero are changed according to relative directions formed by respective pixels and the data interpolating position;

FIG. 7 is an explanatory graph of a sinc function; and

FIG. 8 is an explanatory graph of data interpolation using the sinc function.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

A data processor of an embodiment to which a two variable data interpolation system of the present invention is applied is characterized in that the data processor performs interpolation processing between respective discrete data positioned at constant intervals in a two-dimensional space by using a sampling function that can be differentiated finite times and has values of a local support. Hereinafter, a data processor according to an embodiment will be described in detail in reference to drawings.

FIG. 1 is a block diagram showing the configuration of a data processor of this embodiment. The data processor shown in FIG. 1 performs interpolation processing on the basis of discrete data in a two-dimensional space that is inputted, and includes a discrete value extracting section 10, a sampling function operating section 20, and a convolution operating

section 30. Hereinafter, it is assumed that image data consisting of, for example, saturation data of an image, color data, and the like is considered as the discrete data in the two-dimensional space.

The discrete value extracting section 10 as the discrete data extracting unit extracts and holds a plurality of pixel data, which is included in a predetermined range around a data interpolating position that becomes an interpolation object, out of pixel data sequentially inputted. FIG. 2 is a graph showing a range of pixel data extracted around a data interpolating position. As shown in this graph, with letting a data interpolating position, which becomes an interpolation object, be p and letting its coordinates be (x, y), an extraction object range is a rectangular region consisting of respective two pixels before and after the data interpolating position in the X and Y directions with the data interpolating position P as a center. Therefore, 16 pixel data included in this range is extracted by the discrete value extracting section 10.

The sampling function operating section 20 calculates distance between pixels, which corresponds to respective pixel data extracted, and the data interpolating position p when the coordinates (x, y) of the data interpolating position p is designated. Furthermore, the section 20 calculates values of the sampling function on the basis of the distance between respective pixels and the data interpolating position. Each value of the sampling function is calculated for each of 16 pixel data outputted from the discrete value extracting section 10.

The convolution operating section 30 performs convolution operation corresponding to 16 points of pixel data by multiplying each of 16 values of the sampling function, which are calculated by the sampling function operating section 20, by each value of pixel data and adding the products. A value obtained by this convolution operation becomes an interpolation value corresponding to the data interpolating position.

Next, data interpolation processing performed by the data processor described above will be described in detail. FIG. 3 is an explanatory graph of a sampling function used in operation in the sampling function operating section 20. A sampling function H(t) shown in FIG. 3 is a function of a local support to which attention is paid on differentiability. For example, the function H(t) can be differentiated only once in the whole region and a function of a local support having finite values, which are not zeroes, when a sample position along a horizontal axis is between -2 and +2. In addition, since being a sampling function, the function H(t) is characterized in that the function H(t) becomes one only at a sample point with t = 0 and becomes zero at sample points with t = ± 1 and ± 2.

It is verified by the present inventor's investigation that a function H(t) fulfilling various conditions described above (a sampling function, one-time differentiability, and a local support) exists. Concretely, with letting a third order B spline function be F(t), such a sampling function H(t) can be defined as:

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$$H(t) = -F(t + 1/2)/4 + F(t) - F(t - 1/2)/4.$$

Here, the third order B spline function F(t) is expressed as:

$$(4t^2 + 12t + 9)/4 \quad ; -3/2 \leq t < -1/2$$

$$-2t^2 + 3/2 \quad ; -1/2 \leq t < 1/2$$

$$(4t^2 - 12t + 9)/4 \quad ; 1/2 \leq t < 3/2$$

The above-described sampling function H(t) is a quadric piecewise polynomial, and uses the third order B spline function F(t). Therefore, the function H(t) is a function of a local support that is guaranteed to be differentiable only once over the whole region. In addition, the function H(t) becomes zero at $t = \pm 1$ and ± 2.

In this manner, the above-described function H(t) is a sampling function and a function of a local support that can be differentiated only once over the whole region and converges to zero at $t = \pm 2$. Therefore, it is possible to perform interpolation of a value between discrete pixel data using a function, which is differentiable only once, by performing convolution on the basis of respective pixel data using this sampling function H(t).

FIG. 4 is an explanatory graph of calculation of distance between a data interpolating position and each pixel, the calculation being performed by the sampling function operating section 20. Part of an extraction range of the image data shown in FIG. 2 is shown in FIG. 6. In FIG. 6, a point $P_{i,j}$ denotes a value of image data at coordinates (X_i, Y_j), and

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for example, let the coordinates of the data interpolating pixel be $X = X_{i+1} + 0.5$ and $Y = Y_{j+1} + 0.2$.

For example, if calculating distance t1 between a pixel, which corresponds to pixel data $P_{i+1,j}$ with coordinates (X_{i+1}, Y_j), and a data interpolating position, the sampling function operating section 20 obtains a difference ΔX between X coordinates and ΔY between Y coordinates of these two pixels, and calculates the distance t1 on the basis of these values. In case of the pixel data $P_{i+1,j}$, $\Delta X = -0.5$ and $\Delta Y = -1.2$, and hence the distance t1 is:

$$t1 = \{(0.5)^2 + (1.2)^2\}^{1/2}$$
$$= 1.3$$

In addition, it is assumed that each of intervals between adjacent pixels in the X and Y directions is one.

Similarly, if calculating distance t2 between a pixel, which corresponds to pixel data $P_{i+1,j+1}$ with coordinates (X_{i+1}, Y_{j+1}), and a data interpolating position, the sampling function operating section 20 obtains a difference $\Delta X(= -0.5)$ between X coordinates and $\Delta Y(= -0.2)$ between Y coordinates of these two pixels. The distance t2 based on these values is:

$$t2 = \{(-0.5)^2 + (-0.2)^2\}^{1/2}$$
$$= 0.539$$

When obtaining the distance between corresponding pixels and data interpolating positions for respective pixel data, the sampling function operating section 20 calculates values of the sampling function at data interpolating positions corresponding to respective pixels. As shown in FIG. 5, for example, as for the above-described position $P_{i+1,j}$, the

sampling function operating section 20 calculates a value of H(1.3) by substituting distance $t = t1$ (= 1.3) in the sampling function H(t). Similarly, as for the above-described position $P_{i+1,j+1}$, the sampling function operating section 20 calculates a value of H(0.539) by substituting distance $t = t2$ (= 0.539) for the sampling function H(t).

In this manner, if values of the sampling function H(t) corresponding to data interpolating positions for respective image data are obtained, the convolution operating section 30 performs convolution operation by multiplying the values of the sampling function, which are obtained, by image data of respective pixels, $P_{i,j}$ and the like, and adding these multiplication results for 16 image data, and outputs the interpolation value P corresponding to the data interpolating position p.

Like this, the data processor of this embodiment uses a function of a local support that can be differentiated only once over the whole region as a sampling function. Therefore, it is possible to drastically reduce operation quantity necessary for interpolation processing between image data. Owing to this, it becomes possible in image interpolation processing to lighten the load of the processing apparatus and shorten processing time in case of handling huge processing data.

In particular, it is possible not only to reduce operation quantity because it is sufficient to consider only 16 pixel data as processing objects, but also to obtain a value of a sampling function by simple arithmetic of sum of products

because the sampling function is expressed in a simple quadric piecewise polynomial. Therefore, also from this point, it is possible to further reduce the operation quantity.

In addition, since the sampling function used in this embodiment is a local support, there is no truncation error conventionally arising when pixel data that is a processing object is reduced to a finite number. Therefore, this prevents an aliasing distortion from arising, and in consequence, it is possible to obtain an interpolation result with a small error.

Furthermore, the present invention is not limited to the above-described embodiment, but it is apparent that working modes different in a wide range can be formed without departing from the spirit and scope of the present invention. For example, although the sampling function is defined as a function of a local support, which can be differentiated only once over the whole region, in the above-described embodiment, the number of times of differentiability can be set to be two or more. In addition, as shown in FIG. 3, although the sampling function in this embodiment converges into zero at $t = \pm 2$, the sampling function can be made to converge into zero at $t = \pm 3$ or outer values.

In addition, although, in the above-described embodiment, an interval $t0$ where values of the sampling function $H(t)$ become zero is set to be 1 as intervals between adjacent pixels in the X and Y directions, this can be set to be $\sqrt{2}$ as intervals between adjacent pixels in a 45° oblique direction. In this case, it is possible to use the above-described sampling

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function as it is by calculating $H(t/\sqrt{2})$. Alternatively, it can be also performed to set the above-described interval t0 to be a value "a" fulfilling $1 < a < \sqrt{2}$. In this case, it is possible to use the above-described sampling function as it is by calculating $H(t/a)$.

In addition, it can be also performed to change the interval t0, where values of the sampling function $H(t)$ become zero, according to a relative direction between each pixel and a data interpolating position. For example, as shown in FIG. 6, the interval t0 is set according to a direction in which a data interpolating pixel and a pixel to be an operation object of the sampling function are combined. Concretely, let an angle, which the direction in which the data interpolating pixel and pixel to be an operation object of the sampling function are combined forms with a horizontal direction or a vertical direction, be θ ($\leq 45°$), and the above-described interval t0 is set to be $1/\cos\theta$. In this case, it is possible to use the above-described sampling function as it is by calculating $H(t \times \cos\theta)$.

In addition, although, in the above-described embodiment, the sampling function $H(t)$ is defined by using the B spline function $F(t)$, it is possible to equivalently express the sampling function $H(t)$ by using a quadric piecewise polynomial as follows:

$(-t^2 - 4t - 4)/4 \quad ; -2 \leq t < -3/2$

$(3t^2 + 8t + 5)/4 \quad ; -3/2 \leq t < -1$

$(5t^2 + 12t + 7)/4 \quad ; -1 \leq t < -1/2$

$$(-7t^2 + 4)/4 \qquad ; -1/2 \le t < 1/2$$

$$(5t^2 - 12t + 7)/4 \qquad ; 1/2 \le t < 1$$

$$(3t^2 - 8t + 5)/4 \qquad ; 1 \le t < 3/2$$

$$(-t^2 + 4t - 4)/4 \qquad ; 3/2 \le t \le 2$$

INDUSTRIAL APPLICABILITY

As described above, the present invention performs interpolation operation between a plurality of discrete data by using a sampling function that can be differentiated finite times and has values of a local support. Hence, since it is good enough to make only the discrete data included in this local support be an object of the interpolation operation, operation quantity is few, and no truncation error arises. Therefore, it is possible to obtain an interpolation result having a small error.